Report Progress

Field of research: Mathematical Analysis, Functional Equations and Inequalities.

Project title: Comparison and characterization problems in general classes of means.

Postdoctoral Researcher: Amr Zakaria Mohamed AbdelHamed.

Host professor: Prof. Zsolt Páles.

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Period Covered: 1/9/2020 to 30/6/2021.

1. Aims of the Research Study (as originally proposed in the Postdoctoral application)

The aim of the research plan lies in the field of Functional Equations and Inequalities and it has further applications to mathematical analysis. In particular, it investigates the comparison, the equality, the homogeneity, the invariance and the characterization problems in general classes of means. We had reached significant progress in answering several questions and, as a result within these 10 months, one paper has been published, two papers are under evaluation, and there is another paper under preparation. Moreover, I gave three online talks. I am still actively working under the supervision of Prof. Zsolt Páles on the further research problems which have been listed in my new scholarship application for 2021-2022.

2. Summarize the main progress achieved

The aim of the paper [1] was to study the equality problems of Bajraktarević means and of Cauchy means. Under at most sixth-order differentiability assumptions for the unknown functions, we obtained several necessary conditions in terms of ordinary differential equations for the solutions. Moreover, we introduced eight equivalent conditions for each equality problem.

Paper [2] was motivated by the result of H. Alzer and S. Ruscheweyh published in 2001, which states that the intersection of the classes of two-variable Gini means and Stolarsky means is equal to the class of two-variable power means. The two-variable Gini and Stolarsky means form two-parameter classes of means expressed in terms of power functions. They can naturally be generalized in terms of the so-called Bajraktarević and Cauchy means. Our aim was to show that the intersection of these two classes of functional means, under high-order differentiability assumptions, is equal to the class of two-variable quasiarithmetic means.

The goal of the paper [3] was to introduce an extension of the celebrated Abel–Liouville identity in terms of noncommutative complete Bell polynomials for generalized Wronskians. We also characterized the range equivalence of n-dimensional vector-valued functions in the subclass of n-times differentiable functions with a nonvanishing Wronskian.

3. Academic Output

3.1. **Published publication.**

[1] L. Losonczi, Zs. Páles, and A. Zakaria, *On the equality of two-variable general functional means*, Aequationes Math. **95** (2021).

3.2. **Submitted publications.**

[2] Zs. Páles, and A. Zakaria, *Characterizations of the equality of two-variable generalized quasiarithmetic means*, submitted to J. Math. Anal. Appl.

[3] Zs. Páles, and A. Zakaria, *An extension of the Abel–Liouville identity*, submitted to Acta Math. Hungar.

3.3. **Publication under preparation.**

[4] Zs. Páles, and A. Zakaria, *On the regularity problem of the equality of two-variable Bajraktarević mean.*

3.4. **List of talks within 1/9/2020–30/06/2021.**

(1) *On the equality problems of generalized quasi-arithmetic means*, Analysis Research Seminar, Institute of Mathematics, University of Debrecen, Online via Webex, June 16, 2021.
(2) *On the equality problems of generalized quasi-arithmetic means*, Functional Equations and Inequalities Seminar, University of Silesia, Poland, Online via Zoom, May 31, 2021.
(3) *On the intersection of the classes of Bajraktarević and Cauchy means*, Workshop dedicated to the 80^{th} birthday of László Losonczi, University of Debrecen, Hungary, Online via Webex, February 17, 2021.